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Tools for extracting new physics in events with missing transverse momentum

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We review tools that have been developed in recent years to maximize our ability to discover and characterize new physics appearing in LHC events with missing transverse momentum.

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1. Introduction, Aims and Scope

The 7 TeV run of the Large Hadron Collider (LHC) is now well underway and a plethora of searches for new physics has already been carried out.

This short review concerns searches for new physics in LHC events with missing transverse momentum.^a Many of these searches are based on techniques that have been developed in the last few years, specifically for the advent of the LHC, and the main purpose of this review is to describe some of those techniques and the principles that underlie them.

The review will also cover developments in the strategies that are planned to measure the properties of any new physics that we may be lucky enough to discover. This characterization of new physics is not wholly divorced from the discovery process itself, but the two processes are sufficiently different that they merit separate consideration.

The experimental analyses with which this review concerns itself, like the machine and detectors themselves, are imponderably complicated. This review contains no discussion whatsoever of issues related to hardware, calibration, particle isolation and identification, triggering, jet reconstruction, pile-up, underlying event, and so on. Instead, the starting point will be a set of high-level objects (jets, leptons, missing transverse momentum, etc) in an event, together with their associated measured

^aUnfortunately, the alternative moniker “missing energy”, which is widely used in the literature, may refer to any one of a multitude of different observables;¹ here we insist on using “missing transverse momentum”, whose definition is, we hope, unambiguous.

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momenta and uncertainties thereon. Given these, we shall ask which observable or observables, considered as functions of the momenta (visible or missing), optimize our ability to either make a discovery of new physics or to measure some property thereof.

Even with such a narrow focus, the discussion will, necessarily, be incomplete. The literature is, moreover, large and ever-growing, so that one cannot even hope to catalogue it in a definitive fashion. Rather, the hope is to provide, at least, a useful, if idiosyncratic, introduction to the subject and its methodology.

Before closing this introductory section, it is perhaps useful to spell out the reasons for concentrating on events with missing transverse momentum. One is that methods for dealing for LHC events without missing transverse momentum can often be straightforwardly copied from those used previously at hadron colliders or elsewhere.^b To take an example which we shall discuss in more detail below, in searching for new resonances it suffices to “blindly” compute invariant mass distributions of some combination of final state momenta. But in events with missing transverse momentum, we shall argue that the optimal observable strongly depends on the details of the signal dynamics. So, even though we have, by now, a great deal of experience in studying missing transverse momentum events (for example, in the discoveries of the W -boson and top quark at CERN and Fermilab respectively) , we still have to work hard to be sure that we have optimized our ability to discover physics at the LHC whose nature is not yet known to us.

A second reason, of course, is that many interesting scenarios of new physics, both within the Standard Model (SM) and beyond it, do predict missing transverse momentum events at the LHC. Indeed, we can expect to see missing transverse momentum whenever a neutrino is produced in the decay of a Higgs boson. Moreover, we hope to be able to produce new, invisible particles at the LHC, most notably Dark Matter.

A third reason is purely pragmatic. Data is produced in LHC collisions at such a rate that one must either throw most of it away at random with a “pre-scaled” trigger, or one must trigger on some characteristic. One suitable characteristic is missing transverse momentum, not least because its presence tells us that the event cannot be some tedious QCD process, which overwhelmingly dominate the total cross-section. Unfortunately, most of the time such events will be QCD, but badly measured, and indeed a large part of the remaining work will be to separate this background from a signal of events with genuine missing transverse momentum.

The outline is as follows. In the next Section, we discuss the issue of finding an optimal observable and why there is a necessity to do so in the first place. In Section

^bIt is certainly true, however, that the LHC, which operates in a previously uncharted regime of energy and luminosity, creates challenges and opportunities of its own, even in events without missing transverse momentum. As a simple example, the fact that typical partonic collision CM energies at the LHC greatly exceed the masses of SM particles leads to an abundance of events with highly-boosted objects in the final state, whose decay products are well-collimated. This has led to the development of jet substructure techniques, reviewed in Ref. 2.

3, we consider one approach to finding such an optimal observable^c in the context of what is perhaps the most simple example, namely leptonic decays of the W -boson. We then generalize the same methodology, which essentially amounts to the assumption that the most important distinguishing feature of the signal (compared to backgrounds) is its kinematics, to more complicated situations, including decays of a Higgs boson to W -bosons or τ -mesons and pair production of supersymmetric partners of SM particles.

The $h \rightarrow \tau\tau$ example is particularly appropriate for the purposes of our general discussion, since a number of more sophisticated approaches seem to be possible. Even focussing only on kinematics, it is possible to fully reconstruct events at the LHC, up to a discrete ambiguity, meaning that one can, essentially, carry over the same bump hunting methods used for events without missing transverse momentum. But $h \rightarrow \tau\tau$ searches are also interesting because they give real-world examples of analyses which try to go beyond kinematics and try to use dynamical information to discriminate signal from background. We discuss analyses proposed or in use by both ATLAS and CMS in Section 4. We then go on to consider how assumptions about the dynamics may help in SUSY searches, in a variety of ways.

Finally, we turn to the issue of measuring the properties of new physics, such as masses, spins, and couplings. There already exist reviews covering these subjects, so our discussion will be brief. We also discuss more generally the problem of how we can make inferences about Dark Matter on the basis of LHC measurements and how best to characterize new physics in its nascent stages.

2. Optimal Observables

To begin with, one might wonder why there is any need to devote a review to discussing optimal observables for events with missing transverse momentum. Indeed, the question of ‘Which observable is optimal?’ was answered for us long ago by the statisticians. The answer is that the optimal observable is given by the likelihood function. In particle collisions, this amounts to the matrix element for a particular scattering process, including signal and background contributions, convoluted with the appropriate detector response and marginalized with respect to quantities that go unmeasured.

Unfortunately, this observable is nigh-on impossible to compute in practice: not only is our modelling of the matrix element (for either signal or background) and detector response insufficient, but also the computational effort required to carry out such an analysis at the LHC is inconceivable.

Even then, one might wonder why there is any need for phenomenologists to scratch their heads trying to find an optimal observable. At least when one is dealing with high-level objects after event selection, any observable may be written as a

^cNeedless to say, the notion of which observable is optimal depends on one’s exact criterion for what is optimal.

function of only a few basic observable momenta. So one might object that again the statisticians have already done the hard work in designing multivariate analyses, such as neural networks, boosted decision trees, etc, which will find the optimal observable for us, once we have trained them up sufficiently.

There are two counter-objections to this objection. The first, as de Rújula and Galindo³ so charmingly put it, is that there are times when one would like to have the “pleasure of understanding with use of one’s own neural network” in order to have a clear picture of what is going on. The second is that the sophisticated multivariate analyses on the market are perhaps not so sophisticated, in that they typically only form linear combinations of the initial observables. As such, they miss variables constructed, for example, as products or ratios of the initial observables. Indeed, one does not have to look too hard to find examples in the recent literature where humans have succeeded in finding useful observables where multivariate methods have failed.⁴ So, *nolens volens*, there is still work for us to do.

Even though the true likelihood function is unattainable, it does suggest a useful way to approach the problem of finding an optimal observable. Indeed, imagine that one is able to identify which aspect or aspects of the physics most affects our ability to make a discovery or a measurement in a particular channel. One could then try to find the observable that best takes account of this aspect. In the next Section, we shall develop this idea in the context of several examples in which it is the differing kinematic properties of the signal and background which are assumed to be paramount.

3. Kinematics

We have already mentioned how one might search for a resonance in events without missing transverse momentum. The tried and trusted way to do so is to plot the invariant mass of some combination of final state particles and to look for a bump. This is a useful thing to do, not only because Lorentz-invariance guarantees that signal events will pile up at the resonance mass (increasing the discovery potential), but also because it is rather hard to imagine effects coming from smoothly-varying backgrounds and detector response which could give rise to a bump.^d Thus, even with almost complete ignorance about the nature or size of the backgrounds and detector response, one might confidently claim a discovery if a bump were seen.

In this example, the aspect of the physics which is most important is the (assumed) presence of a narrow resonance peak in the invariant mass distribution of the signal and so the natural choice of observable is the invariant mass. The assumption is optimal^e not only for discovery, but also for measuring the mass of the resonance,

^dIn contrast, it is rather easy to obtain features like “tails” and “shoulders” in distributions by superposing simple background components and caution should be exercised in claiming discoveries on the basis of such features.

^eWe stress that we have not proved that the observable is optimal, not least because we have not defined what we mean by “optimal”. We use it in a colloquial sense.

since signal events pile up at the place where the measurement is performed.

In a sense, this way of proceeding is akin to following the likelihood approach, albeit in a cavalier fashion. In effect, one is making the drastic approximation that “the physics” is encapsulated by the chosen aspect (kinematics) and nothing else and then making a crude guess as to the resulting likelihood variable.^f

It is important to note that making drastic approximations of this kind cannot invalidate the subsequent analysis, even if the approximations made are badly violated in reality. Indeed, the only purpose of the procedure is to define (or rather, motivate) an observable. But some observables are better than others and so the price that one pays for a poor approximation is an observable that is far from being optimal.

So, one can already make a lot of progress just by using kinematic features of the signal, choosing an observable whose signal distribution is as “peaked” (or piled-up) as possible. Of course, in order for this to be a useful course of action, one should take care that the background not be equally sculpted at the same time!

This idea, as we will see, generalizes in a very natural way to events with missing transverse momentum. When one (or more) of the final state particles is (or are) invisible, we cannot hope to construct a resonance bump in an invariant mass, since the invariant mass is no longer observable. But we can try to construct an observable for which the signal is as peaked as possible. To do so, let us begin with the case where all final state particles are visible, for which the invariant mass observable is always equal to the resonance mass. To put it another way, the observable is bounded both above and below by the mass of the resonance. With one or more final state particles invisible, we shall not be able to find an observable which is bounded both above and below, but we shall be able to find observables which are bounded above or below. By finding the observable which gives the strongest upper or lower bound, we obtain an observable for which the signal is as peaked as possible, and which may be considered optimal in the sense of being discoverable in the presence of a smoothly (and slowly) varying background. What is more, if the location of that peak (or other sharp feature) is strongly correlated with some physical property of the signal (such as the mass of the resonance), then it may be that the same observable is optimal for the purpose of that measurement.

We now consider an example in which this observable is nothing other than the transverse mass.

3.1. *Bounding Variables*

3.1.1. *W and W' decays*

Consider decays of a W -boson into a charged lepton and an invisible neutrino. One way to define the transverse mass, m_T ,⁵ is as the observable that provides the

^fPresumably, one could follow this recipe in a rigorous fashion, but we shall not do so here.

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greatest lower bound on the (*a priori* unknown) mass of a resonance which decays into some combination of visible and invisible final state particles.

The classic example is the decay of a W -boson into a charged lepton and an invisible neutrino, for which the above prescription yields

$$m_T^2 = 2(|p_T||\not{p}_T| - p_T \cdot \not{p}_T), \quad (1)$$

where p_T is the transverse momentum of the charged lepton and \not{p}_T the missing transverse momentum.

In this way, m_T is seen to be the optimal variable (in the sense described above) to discover the W -boson (although it was not employed in that way in the original analysis^{6,7}). In fact, m_T is even better than may appear on the basis of the above considerations. Why? Because m_T is able to separate the signal from some of the backgrounds. Indeed, there are plenty of ways to arrive at an event containing a charged lepton, missing transverse momentum and zero or more jets and each of these is a source of background. For example, every time one produces a B -meson, then ten per cent of the time these decay leptonically, resulting in precisely the same final state. However, for these events, m_T cannot exceed the mass of the B -meson, namely a few GeV. Thus, m_T has the additional advantage of providing a separation between the signal events (which may take values all the way up to m_W) and background events coming from leptonic B -decays, which lie below m_B , if well-measured.

If, instead of a charged lepton, the visible particle in the final state had been a jet, then another dominant source of background would come from events in which the energy, but not the direction, of the jet had been mismeasured, leading to apparent, fake, missing transverse momentum, aligned with the jet. But for such a configuration, m_T is given, roughly, by the jet mass, which again is typically much smaller than m_W .

It is important to note that one does not have a theorem to the effect that there is a clean separation between the signal and all backgrounds. In typical final states there will be many background components and not all of them will enjoy this property. But this does not render the above observations worthless. The point is that, in any given analysis, one expects that certain background components will be well understood while others will be less so. One can also, therefore, try to proceed by finding an observable which is insensitive to the background with the largest uncertainty.

One can immediately apply these ideas to searches for new physics at the LHC, namely to searches for a W' -boson. Now, the dominant background comes from the W -boson itself, but again one is guaranteed a clean separation of signal and background, at least for well-measured events, since for the background events $m_T < m_W$. This strategy is exactly that which is employed in recent LHC searches.^{8,9}

As an aside, the fact that m_T gives the greatest lower bound on the parent particle mass means that it contains all of the information which follows from the kinematic constraints (namely conservation of energy-momentum and the mass-shell

constraints) alone.¹⁰

3.1.2. Di-leptonic Higgs decays via WW

The above approach generalizes straightforwardly to decays of a resonance into multiple visible particles, with the observable m_T in (1) being replaced by

$$m_T^2 = m_v^2 + 2(\sqrt{p_T^2 + m_v^2}|\not{p}_T| - p_T \cdot \not{p}_T), \quad (2)$$

where p_T is now the transverse momentum of the visible particle system and m_v is its measured invariant mass.

For decays with multiple invisible particles, things are a little more subtle. Naïvely, one would define the transverse mass by

$$m_T^2 = m_v^2 + m_i^2 + 2(\sqrt{p_T^2 + m_v^2}\sqrt{\not{p}_T^2 + m_i^2} - p_T \cdot \not{p}_T), \quad (3)$$

but this is not an observable, since the invariant mass of the invisible system, m_i , is not observable. However, since m_T in (3) is a monotonically increasing function of m_i , one immediately sees that the best lower bound on the mass of the decaying resonance is given by replacing m_i by its minimum value, namely the sum of the invisible particle masses, if known, or, if not, by its minimum possible value, namely zero.

An immediate application¹¹ (see also Refs. 12, 13) is to decays of a heavy (above 135 GeV or so) SM Higgs boson into two charged leptons and two neutrinos via (possibly virtual) W -bosons. The resulting observable, called m_T^{true} in 11 is just given by the right hand side of (2) and is central to state-of-the-art analyses carried out by ATLAS¹⁴ and CMS¹⁵. In more detail, ATLAS, for example, performs¹⁴ a cut-and-count analysis in the interval $0.75m_h < m_T^{\text{true}} < m_h$, for varying Higgs mass m_h (after other cuts). In accordance with the arguments given above, this maximizes the signal contribution in the cut region compared to the dominant background (which comes from continuum WW production.)^g

In fact, for Higgs decays in this channel, one can do even better. For m_h above $2m_W$, both W -bosons will typically be produced on-shell, whilst even for m_h below $2m_W$, one or other W -boson will typically be produced on-shell, at least for Higgs masses large enough to give sizable cross-section times branching fraction in the WW channel.

This observation can be used in the following way. One can define two new observables, m_T^* and m_T^{bound} , as the ones that give the greatest lower bound on the mass of the (Higgs) signal resonance, subject to the condition that either one or

^gPrevious analyses used the opening angle between the two charged leptons, $\Delta\phi_{ll}$, as the figure of merit. This was based on the observation that, for a spinless resonance (such as the SM Higgs) of mass $2m_W$ decaying (at threshold) to WW , conservation of angular momentum forces the two charged leptons to be parallel. Away from threshold, the two charged leptons remain well-correlated. This is a neat example of an analysis in which dynamics, rather than kinematics, has been used to motivate the choice of observable.

other (for m_T^*)¹⁶, or both (for m_T^{bound})¹⁷ of the W -bosons be on shell.^h In both cases, since one is applying additional constraints, one obtains a greater lower bound event-by-event, resulting in an even more sharply peaked signal distribution.

Unfortunately, one cannot give an explicit, analytic form for these two variables, but they can be straightforwardly computed using an algorithmic computer code. Perhaps unsurprisingly, each gives a significant improvement only in the region in which the assumptions that are made in the definition hold to a good approximation.

3.2. Higgs decays via $\tau\tau$

The variable m_T^{bound} was first introduced in the context of Higgs decays to $\tau\tau$,¹⁸ which each, in turn, decay either leptonically, producing two neutrinos, or hadronically, producing only one neutrino. One apparent disadvantage of this variable for the $\tau\tau$ channel is that the measured momenta must satisfy the condition $m_{T2} < m_\tau$ in order that m_T^{bound} be well-defined. This is guaranteed for well-measured signal events, but in the presence of detector mismeasurements, events may easily fail to satisfy this condition. Indeed, naïvely, a mismeasurement of \not{p}_T by $\delta\phi_T$ would lead to a similar mismeasurement in m_{T2} ; since $\delta\phi_T$ typically far exceeds m_τ at the LHC, one would expect the condition would rarely be satisfied. In reality, the situation is not so bad,¹⁷ but still one finds that m_T^{bound} does not exist for about a third of events in simulations, leading to a significant reduction in statistics.ⁱ

There are other approximations which might be made for $\tau\tau$ decays, resulting in other observables. For now, we restrict our attention to those which are solely concerned with kinematics, deferring consideration of dynamics to the next section.

Firstly, given existing limits from LEP on the mass of the Higgs boson, it is likely that the produced τ s will be significantly boosted in the lab frame, in which case one could assume that their decay products are collinear.¹⁹

If one makes this assumption, then in fact there are as many constraints as unknowns, such that the momenta (and the invariant mass of the $\tau\tau$ system) can be reconstructed unambiguously. To wit, the two unknown proportionality constants between the visible and invisible momenta in each τ decay can both be determined from the two missing transverse momentum constraints. So one ends up not with a lower bound on the invariant mass of the $\tau\tau$ system, but its precise value, assuming that things are well-measured.

In practice, this collinear approximation seems to be not such a good one: the presence of finite detector resolution and acceptance results in solutions with unphysical energies in a significant number of events (as many as half in simulations carried out by the experimental collaborations).

^hIn retrospect, history might have been kinder to us if m_T^{true} , m_T^* , and m_T^{bound} had been called $m_T^{W^*W^*}$, $m_T^{W^*W}$, and m_T^{WW} , respectively.

ⁱOne can, of course, resort to using a different observable, such as (2) for these events, so they need not be discarded.

An alternative kinematic strategy for hadronic τ decays is to use the extra information which is available in such events from the location of the secondary (τ decay) vertices.²⁰ The best measured attributes of these are their impact parameters, namely the displacements of the secondary vertices measured in a direction perpendicular to the visible decay momenta. The invisible momentum in each τ decay must lie in the plane of the impact parameter and the visible momentum, leading to a quartic equation (and a fourfold ambiguity) in the reconstructed Higgs mass. The problem arises of how to deal with the resulting discrete ambiguity (which in other cases may be compounded by a combinatorial ambiguity as to which of multiple, indistinguishable final state particles should be assigned to which decay or part of a decay. A similar situation (but for cascade decays of supersymmetric particles) was studied in Ref. 21. There it was empirically observed that the wrong solutions appeared to be correlated with the right solution. An explanation was given in Ref. 20. In the specific case of τ decays considered above, for example, it was pointed out²⁰ that, in the limit that the Higgs mass becomes large compared to the mass of the τ , the quartic equation reduces to a linear equation, with the four solutions coalesced in a single solution. In the real world, we are slightly away from the limit, but the solutions remain close together. The fact that the wrong solutions are correlated with the right one means that one can simply retain all solutions (indeed in Ref. 20, it was pointed out that it is even useful to retain complex solutions, to increase the available statistics): the correlation means that the “signal” (in the form of the correct solution) will not be overwhelmed, but rather will be reinforced, by the “background” coming from the wrong solutions.

3.3. Supersymmetry and Dark Matter

These ideas can all be extended straightforwardly to events with new invisible particles, such as the LSP of a supersymmetric theory or another Dark Matter candidate, rather than common or garden neutrinos. If one is only interested in kinematics, the only relevant property of an invisible particle is its mass. As we shall see below, this poses an important complication when it comes to making mass or other measurements, since it introduces another unknown to be measured. But for discovery purposes, the only effect of a massive invisible is that the scale of the visible particles in the final state is set, roughly speaking, by the mass difference between the parent particle (or particles) produced in the hard process and the invisible daughter particles. This is significant for search strategies of the type discussed above, which aim for a kinematic separation between signal and background, since the smaller this mass difference becomes, the more the signal overlaps with the SM backgrounds. So even heavy (but approximately degenerate) new particles may be difficult to discover in this way. To make progress in such a scenario, one would need to use information about the dynamics as well. We postpone discussion of this to §4.

The second important complication of supersymmetric theories is that the parent

particles are pair produced, with each decaying to one (or more) invisible daughter particles. Thus, rather than assume, as we have done above, that the observed missing transverse momentum can be attributed to a single decay, we should only assume that it is partitioned in some way between the two decays.

Once we do so, we can again blindly follow our procedure of asking which observable gives the greatest event-by-event lower bound on the mass of the decaying parents. The resulting variable, whose definition remains algorithmic except in special cases,^{22,23,24,25} goes by the name of m_{T2} .^{22,26} Just as for m_T , m_{T2} gives us a variable which is as “peaked” as possible for the signal, leading us to hope that a significant signal excess can be identified on top of backgrounds. Even better, just like m_T it has properties which serendipitously ensure a good separation between the signal and several of the dominant SM backgrounds, provided that the mass difference between the parents and invisible daughters is large.²⁷ This makes our understanding of the backgrounds that appear in the signal region and the systematic uncertainties thereon more robust, enabling a discovery or exclusion to be made even in the absence of a sharp feature in the signal distribution.^j Both general-purpose collaborations have now presented SUSY searches employing m_{T2} .^{28,29}

Again, it follows that m_{T2} captures all of the information that is contained in the energy-momentum conservation and mass-shell constraints alone.^{30,31,10} This will be important in our discussion of mass measurement, in that if invisible particle masses cannot be measured using m_{T2} , then they cannot be measured at all using kinematics alone. Happily, they can.

Once particles are pair produced, we must consider the ambiguity that arises in deciding which of the visible particles in the final state should be grouped together and associated to the decay of the parent. This problem is, of course, most acute when the decays are identical. Again, one can take this ambiguity into account in designing the discovery observable, by asking which observable gives the greatest lower bound on the parent mass, after minimizing in addition with respect to the possible combinations. The resulting variable is called m_{TGen} .³² A different procedure has been suggested in Ref. 33.

Finally, one can use a similar trick to deal with ambiguities that arise in deciding which jets are likely to have come from the final state and which from radiation in the initial state (insofar as these are well-defined notions).³⁴

3.4. *Other applications*

This simple but far-reaching idea, of optimizing discovery potential by finding the observable that provides the strongest bound on the mass of a new particle has also been applied in the context of searches for charged Higgs bosons in top quark

^jUnfortunately, one of the sad facts of life is that the more invisible particles one has in an event, the less sharp is the signal, essentially because the available energy in a collision can be shared out among the particles in many more ways.

decays³⁵ and for pair-produced third-generation leptoquarks³⁶ decaying to a $b\nu\tau$ final state.³⁷

3.5. Singularity Variables

The kinematic ideas discussed above could, in principle, be applied to events with arbitrarily complicated topologies. We have already seen, however, that even in relatively simple cases, such as $h \rightarrow WW \rightarrow 2l2\nu$, it is not possible to find an explicit, analytic expression for the observable that encodes all of the information in kinematics.

Kim³⁸ has noted that the notion of a bounding variable arises because of end-point singularities that appear when one projects the full phase space of some event topology onto its observable subspace (a simple example was given in Ref. 39, in which the full phase space is a sphere and the observable phase space is a disk, singular on its edge). He has noted, furthermore, that these singularities show more general behaviour than simply endpoints (such as cusps^{40,41}); these more general features may also provide a way to identify a signal on top of a smoothly-varying background. Once the singularities corresponding to a given signal topology have been found, one may try to construct an optimal observable in the region of the singularity, according to various criteria of optimality. De Rújula and Galindo³ have attacked this problem in the simplest case of W -boson decays, defining observables which are optimal in a statistical sense.

4. Dynamics

All of the methods discussed above use only the kinematic properties of the signal and backgrounds. We now discuss analyses which try, either separately or in addition, to exploit what is known, or may be assumed, about the dynamics.

4.1. Higgs decays via $\tau\tau$, again

For $h \rightarrow \tau\tau$ searches (either in the SM or MSSM), both ATLAS⁴² and CMS⁴³ have employed more sophisticated approaches using *ad hoc* likelihood functions. Neither of the approaches implemented so far uses information from the secondary vertices, such that the kinematics of the signal are underconstrained. The analyses are similar in that they both allow the unconstrained parameters, namely the momenta and invariant masses (for hadronic decays) of the invisible systems in each τ decay to vary, subject to the mass shell and missing transverse momentum constraints. The Higgs mass is then reconstructed by maximizing a likelihood function on this space, whose nature differs in the two analyses. The ATLAS likelihood function⁴⁴ contains two ingredients. The first is a probability density function for the τ decay angles, which is obtained by fitting a simulation of $Z \rightarrow \tau\tau$ decays. The second ingredient is a Gaussian smearing of the measured missing transverse momentum.⁴⁵ CMS are more reticent about the details of their likelihood function; we are at least told that

it “[takes] into account the tau-decay phase space and the probability density in the tau transverse momentum, parametrized as a function of the tau-pair mass”.⁴³

4.2. *Supersymmetry and Dark Matter, again*

There have been several, rather different, efforts to use dynamical information to optimize discovery of theories with pair production, such as supersymmetry. The most basic of these continue the spirit of the kinematic discussion above, but either supplement or substitute it with dynamical assumptions.

As an example, one might make the dynamical assumption that, because parton distribution functions fall off steeply with increasing x , parents will tend to be pair produced at rest relative to one another in their CM frame, so as to minimize the resulting \hat{s} . If one makes this assumption, then the natural observable is defined in Ref. 46, where it is denoted M_R . Since pairs of squarks are produced in a p -wave state at leading order in QCD in quark-antiquark collisions, it is clear that this assumption cannot be valid in certain cases. But, as discussed above, this does not render the observable invalid; rather, it only makes it possibly sub-optimal.^k

It certainly is true, however, that pairs of parents are produced back-to-back in their CM frame. If one made, furthermore, made the dynamical assumption that the CM frame of the parents was dominantly boosted only longitudinally with respect to the colliding beams, then the optimal observable might be one which was invariant under both longitudinal boosts of the CM frame of the parents and back-to-back boosts of the parents in their CM frame. Indeed, one would expect the bulk of signal events to occur at the same value of such an observable (in exactly the same way that Lorentz invariance guarantees a peak in the invariant mass in decays without missing transverse momentum). The appropriate variable goes by the name of m_{CT} .⁴⁷

These approaches could be applied to arbitrary pair decay processes. For longer cascade decay chains, we have seen that it is kinematically possible to reconstruct events. However, one might wonder how reliable such a reconstruction is in the presence of detector mismeasurements. One way to deal with this is to construct a likelihood which takes the detector response into account.^{48,49,50}

A completely different approach deals with the aforementioned difficulty of discovering SUSY using naive kinematic observables if the superpartner spectrum is roughly degenerate, or “compressed”.

At least if the superpartners are heavy, one important feature of the signal is that it leads to a harder pattern of radiation in the initial state than what is typically obtained for SM backgrounds. Thus, the optimal way to discover such scenarios may be to focus on events with more jets than expected from the signal final state and with large missing transverse momentum, arising when the parents recoil strongly against the initial state radiation.^{51,52} By incorporating a number

^kRef. 46 also introduced a modified variable M_{R^*} , designed to deal with this issue.

of selection cuts, one can achieve a good coverage of the signal parameter space, without the need for sophisticated observables.⁵³ One can also try to tag initial state radiation explicitly.⁵⁴

Yet another direction is to focus instead on the dynamics of the background rather than that of the signal. So far, efforts have been directed at the dominant background of mismeasured QCD events. Here, the presence of the background is purely a detector effect of which one can hope to build up a good understanding. As a simple example, the fact that the largest mismeasurements come from jet energies (rather than jet directions) means that one can suppress much of this background by cutting on an observable with known properties under such mismeasurements. One such observable, α_T ,⁵⁵ works well for di-jet events, though the signal rejection is high. More challenging are events with multiple jets, since our understanding of the SM background prediction is very poor. ATLAS⁵⁶ has performed a search in events with between six and eight jets in which the SM background is determined by extrapolation from measurements in events with fewer jets. The key observable is $|\not{p}_T|/\sqrt{H_T}$, where H_T is the scalar sum of the transverse momenta of all jets within some fiducial region. This ratio measures the missing transverse momentum relative to the resolution due to stochastic variations in the measured jet energies that are assumed to follow the central limit theorem. It is then expected that the distribution of this observable will be roughly invariant under changes in the jet multiplicity, such that the background in the event sample with high jet multiplicity can be estimated from the measured distribution in the low jet multiplicity sample, which is assumed to be background dominated. A similar search was carried out by CDF,⁵⁷ using a cut on this variable. CDF's search also employs a technique in which the total missing transverse momentum is compared with that observed using the charged particle spectrometer only, the rationale being that these should be aligned for signal events, but may be anti-aligned for background events arising from calorimeter jet energy mismeasurements.

Finally, one could attempt a more rigorous likelihood method using genuine matrix elements for signal process. Though this has not been explored in the context of discovery, a study of the prospects for mass measurement has been carried out.^{58,59}

5. Measuring New Physics

We now turn to a discussion of methods that have been developed to measure the properties of new physics, once it has been discovered. As we remarked in the introduction, it is clear that the process of making measurements is not wholly divorced from the process of making a discovery, just as the process of excluding models currently underway at the LHC depends on where in the parameter space one sits. Indeed, many of the observables and strategies proposed for making measurements coincide (or nearly coincide) with discovery strategies. This is hardly surprising, in that observables which do a good job of distinguishing signals from backgrounds will also facilitate making clean measurements.

Furthermore, the topics of mass and spin measurements have already been reviewed in the recent literature.^{60,61,62} Our discussion will, therefore, be brief.

5.1. *Mass measurements*

The first measurement priority will be to establish the kinematic properties of the signal, namely the masses of any new particles. Just like the discovery process, this is complicated by the fact that kinematic information is lost in events with invisible particles.

It has long been known (beginning with the pioneering study of endpoints in invariant mass distributions in Ref. 63) that some information about particle masses (roughly speaking, mass differences) can be obtained using kinematics alone.

In events involving cascade decays, with on-shell intermediate states, it may be possible, as discussed above, to reconstruct events. This approach was pioneered in Ref. 64 and has been developed in Refs. 65, 48, 21, 66, 49, 50, 20. The business of counting the various unknowns and constraints is straightforward and we shall not describe it here. As an example of how many constraints is required, for pair decays one needs two, on-shell intermediates in each chain in order to be able to reconstruct the masses.

A crucial question is whether masses can be measured absolutely in decays with fewer on-shell intermediates, using kinematics alone. A definitive answer to this has been provided as follows: when a parent particle (or a pair of identical parent particles) decays into a system of visible daughter particles and a system of invisible daughter particles, there is enough information in kinematics to measure the mass of the parent and the sum of the masses of the invisible daughters, even when there are no on-shell intermediates.^{67,68,69,23} In particular, if a decay process involves only a single invisible dark matter particle, then the mass of that particle, can, in principle, be measured, no matter what decay produces that particle.

What has not been answered definitively is the question of how best to measure those masses, or indeed whether it can be achieved in practice. Of course, the answer to this question depends to a large extent to the nature of the signal and many different methods have been put forward. These have been painstakingly reviewed in Ref. 62, to which the reader is referred for details.

Regrettably, along with this plethora of methods has come a *smörgåsbord* of different observables with confusingly similar names. Ref. 1 counted over fifty different variables in the literature which go under the name of “transverse mass”, with or without further qualification. This state of affairs is probably unavoidable; in any case, there would seem to be an onus upon authors and workers in the field to take care over the definition of observables.

5.2. *Spin measurements*

Spin measurements are challenging even in the absence of missing transverse momentum, because spins (being the generators of rotations) manifest themselves in

angular effects. In order for such effects to be present in a decay, for example, one first needs to create a preferred axis of some kind. So to have observable spin effects often requires a complicated event topology. For example, in a cascade decay, in order that non-trivial angular correlations exist, one requires at least that the intermediate particle in question be polarized, and if a fermion, that its decay be chiral.⁷⁰

Things are even more difficult given the presence of missing transverse momentum, because one needs to be able to reconstruct reference frames in order to be able to measure angles explicitly. One way to circumvent this problem is to focus on Lorentz-invariant quantities, which take the same value in all frames. Alternatively, one needs to reconstruct events, which can only be achieved for certain topologies, as described above.

There are, nonetheless, many ideas on the market, most of which apply only to a specific decay topology. They are reviewed in Refs. 60, 61.

6. Other measurements

Besides measuring masses and spins, one would eventually like to measure all of the parameters appearing in the lagrangian that describes physics at the TeV scale. These include the gauge and global charges of new particles (with respect to whatever symmetry group is present at that scale), as well as the coupling strengths of the interactions that are permitted by that symmetry group. The method by which one might achieve this is, of course, greatly contingent upon the nature of the new physics and relatively little work has been appeared in the literature.

To give just one example of what has been done, Ref. 71 explored in detail the procedure by which one might measure couplings and mixing angles, in addition to masses and spins, in cascade decays using invariant mass observables.

We now describe a rather different measurement that has been proposed recently,⁷² namely to count invisible particles in events.

6.1. Counting invisible particles

If we are fortunate enough to discover new physics associated with missing transverse momentum at the LHC, there will surely be a great temptation to identify it with the Dark Matter that dominates the matter in the cosmos.

The question arises, though, of how to make that identification concrete. The holy grail would, presumably, be to compute the expected relic density using fundamental parameters of the Lagrangian measured at the LHC or in other terrestrial experiments and to compare with the observed cosmic abundance.

Unfortunately, it would appear that such a comparison is out of reach in the LHC era.⁷³ For one thing, such a comparison would require one to measure a large number of parameters, each one of which would, from the previous discussion, require a herculean effort at the LHC. Indeed, a computation of the relic abundance would require one not only to discover all the new particles appearing at the TeV

scale, but also to determine their masses, spins, and couplings to all other particles. For another, the observed relic abundance implies that it is the weak interaction that is responsible for setting the dark matter density, but weak interaction effects are inevitably overwhelmed by strong interaction effects at a hadron collider and so near impossible to disentangle. As an explicit example, cross sections for processes involving production of new particles followed by their decay are set in the narrow width approximation by the product of the production cross section and the branching fraction for the decay, but the production cross-section will usually be dominated by the strong interaction.

Ref. 72 made the rather different suggestion of trying to gain information about a Dark Matter candidate by simply counting invisible particles in events with missing energy. Indeed, multiple production would already constitute evidence for the presence of a symmetry, as required to stabilize Dark Matter on Hubble timescales. Similarly, if one could establish evidence for production of an odd number of invisible particles, one could rule out the simplest (and most commonly employed in models) symmetry, namely Z_2 .

But how can invisible particles be counted in LHC events? The available energy in collisions is, of course, shared out in a random fashion between the particles produced and, as a result, the shapes of various observable distributions vary depending on how many invisible particles are present. As a simple example, if a parent particle is singly produced at the LHC and decays to visible particles and n -invisible particles whose masses may be neglected on collider scales, then the transverse mass (1) has a power-law behaviour near its maximum, with the power given by $n - \frac{3}{2}$.⁷² This result holds independently of the number of visible particles, or indeed of whether there are intermediate particles on-shell. At least in this simple example, it should be rather easy to distinguish between production of a single, invisible particle (for which the distribution diverges near the endpoint) from production of multiple, invisible particles (for which the distribution vanishes).

6.2. *Distinguishing stabilization symmetries*

A related question is how we might try to distinguish between different stabilizing symmetries for Dark Matter. Consider, for example, the two simplest symmetry groups, namely Z_2 and Z_3 . Counting invisible particles provides one way, in that an odd number of invisible particles can only be associated with Z_3 and not Z_2 . Previous work^{74,41,75} attacked this question in a different way, focussing on the observation that models with Z_3 symmetries might contain final states with the same visible particle content, but with different numbers of invisible particles, which would lead to distributions containing two, rather than one, components, which might be picked out in data.

7. Characterization of new physics

Even once we have made a discovery of new physics and possibly measured a limited number of its properties, we shall be faced the question of how best to try to characterize that physics. The holy grail is, of course, to write down the Lagrangian, in which the fundamental principles of symmetry, etc., are as manifest as we know how to make them.

Unfortunately, it seems unlikely that we will succeed in guessing the correct form of the Lagrangian soon after discovering new physics. Indeed, it may not be possible at all to home in on the Lagrangian in the LHC era, let alone to perform precise measurements of its parameters.

But some characterization of the new physics would be invaluable in terms of suggesting where to look for corroborations of discoveries, to perform cross-checks, or indeed to further enhance our knowledge about the nature of the physics in a sort of bootstrap process.

There seem to be two possible approaches. The first is make an educated guess (perhaps on the basis of one's theoretical bias) as to what the exact Lagrangian is, modulo the values of a few parameters. One can then either exclude the model in an increasingly large region of parameter space (in the absence of a signal excess) or attempt to measure those parameters (in the presence of one).

This approach has been very popular, understandably so since it gives one a concrete hypothesis which (one hopes) will eventually be accepted or rejected. One downside is that one's guess must be a good one and, though it is a tale which is too long for us to tell here, our theoretical bias has not in the recent past proven to be a very good indicator for what new physics does lie beyond the Standard Model. A perhaps more pernicious downside is that focussing on a specific Lagrangian can lead to the development of search strategies which, though optimized for that particular Lagrangian, fail spectacularly when one considers other new physics, that is not dramatically different from the original Lagrangian. A salutary example is provided by supersymmetric models with unification of gaugino masses at the GUT scale. The RG evolution leads to a fixed, large (6:1) mass ratio between the gluino and the Bino at the TeV scale and, since the Bino is often the LSP in such models, a fixed signal kinematics. Searches which were developed with such models in mind have proved to be rather ineffective in probing models with compressed spectra.^{51,52}

The second approach avoids this by via a more *ad hoc* strategy of making a coarse guess for new physics in the form of a simplified model^{76,77} which, it is hoped, captures the essence of the physics that is relevant for LHC collisions. This can hardly be described as a novel philosophy, in that physics has been done in much this way for millennia, but the details might be. Typically, a simplified model might contain only a few new degrees of freedom, with masses allowed to float as well as the cross-sections for production and decay. The advantage of such an approach is that it enables a huge variety of models to be rapidly scanned and compared with LHC data; the disadvantage is that such a model is in no way fundamental and it

is hard to see how it could be used to make detailed predictions for new physics phenomena elsewhere, for example in the cosmos. Presumably though, once one had narrowed down the possibilities to a small set of simplified models, one could return to the approach of guessing concrete Lagrangians, for which this can be done.

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